# Mapping Lateral Variations in Apparent Stress: Application of the Coda Ratio Methodology to Southern California Sequences Final Project Report

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#### **Abstract**

The behavior of earthquake source scaling has been the topic of significant debate in the earthquake source community mainly because current methods require substantial path, site, and source radiation pattern corrections that ultimately yield large variance of the apparent stress (e.g., see review by Abercrombie et al., 2006). A new, state-of-the-art methodology, the coda ratio technique, was developed by Mayeda et al. [2007] and provides unprecedented estimates for corner frequency and apparent stress drop, roughly 3 times less variance than conventional methods using a minimal number of stations and events. Within broad regions it is likely that apparent stress varies with location due to lateral variations in regional stress field, rheology, and degree of tectonic activity. The coda ratio methodology has been applied to a variety of tectonic regions and we find clear distinctions between each of them. Preliminary use of local and regional S-wave coda is shown to provide stable amplitude ratios that better constrains source differences between event pairs. The initial study by Mayeda et al. (2007) first compared amplitude ratio performance between local and near-regional S and coda waves in the San Francisco Bay region for moderate-sized events, then applied the coda spectral ratio method to the 1999 Hector Mine mainshock and its larger aftershocks. They found: (1) Average amplitude ratio standard deviations using coda are ~0.05 to 0.12, roughly a factor of 3 smaller than direct S-waves for 0.2 < f < 15.0 Hz; (2) Coda spectral ratios for the  $M_w$  7.0 Hector Mine earthquake and its aftershocks show a clear departure from self-similarity, consistent with other studies using the same datasets; (3) Event-pairs (Green's function and target events) can be separated by ~25 km for coda amplitudes without any appreciable degradation, in sharp contrast to direct waves. In almost all regions we find evidence for nonself similar source scaling. We will use the coda ratio technique to determine 2-D lateral variations in apparent stress in southern California using recent earthquake sequences such as: San Simeon, Northridge, Parkfield, Hector Mine and others as time permits. Our goal is to use this state-of-the-art methodology to constrain apparent stress and map out lateral variations in source scaling.

### Approach

Coda Ratio Processing

Assuming the simple single corner frequency source model [Aki, 1967; Brune, 1970] used in MDAC, the ratio of the moment-rate functions for two events (1 and 2) is given by,

$$\frac{\dot{M}_{1}(\omega)}{\dot{M}_{2}(\omega)} = \frac{M_{0_{1}} \left[1 + \left(\frac{\omega}{\omega_{c_{2}}}\right)^{2}\right]^{p/2}}{M_{0_{2}} \left[1 + \left(\frac{\omega}{\omega_{c_{1}}}\right)^{2}\right]^{p/2}} \tag{1}$$

where  $M_0$  is the seismic moment and  $\omega_c$  is the angular corner frequency  $(2\pi f_c)$  and p is the high frequency decay rate. At the low frequency limit the source ratio shown in equation 1 is proportional to the ratio of the seismic moments  $\left[M_{0_1}/M_{0_2}\right]$ , whereas at the high frequency

limit, equation 1 is asymptotic to  $\left[M_{0_1}/M_{0_2}\right]^{\left[-\frac{p}{3}\right]}$  under self-similarity. If we follow the usual *Brune* [1970] omega-square model and set p=2, the exponent of the high-frequency ratio becomes 1/3. However, it has been proposed by *Kanamori and Rivera* [2004] that the scaling between moment and corner frequency could take on the form,

$$M_o \sim \omega_c^{-(3+\varepsilon)}$$
 (2)

where *e* represents the deviation from self-similarity and is usually thought to be a small positive number. For example, *Walter et al.* [2006] and *Mayeda et al.* [2005] found *e* to be close to 0.5 for the Hector Mine mainshock and its aftershocks using independent spectral methods. For the current study we use the source spectrum portion of the Magnitude Distance Amplitude Correction (MDAC) methodology of *Walter and Taylor* [2001], which allows for the variation of the corner frequency that does not have to be self-similar. For example,

$$\omega_c = \left(\frac{k\sigma_a}{M_0}\right)^{1/3} \text{ and } \sigma_a = \sigma_a' \left(\frac{M_0}{M_0'}\right)^{\psi} \text{ and } \psi = \frac{\varepsilon}{\varepsilon + 3}$$
 (3)

where  $\sigma_a$  is the apparent stress [Wyss, 1970],  $\sigma'_a$  and  $M'_0$  are the apparent stress and seismic moment of the reference event, and  $\psi$  is a scaling parameter. For constant apparent stress,  $\psi = 0$  and e = 0, however, Mayeda and Walter [1996] found  $\psi = 0.25$  for moderate to large earthquakes in the western United States. By using the corner frequency defined in (3) into equation 1, we can apply a grid search to find the parameters that best fit the spectral ratio data.

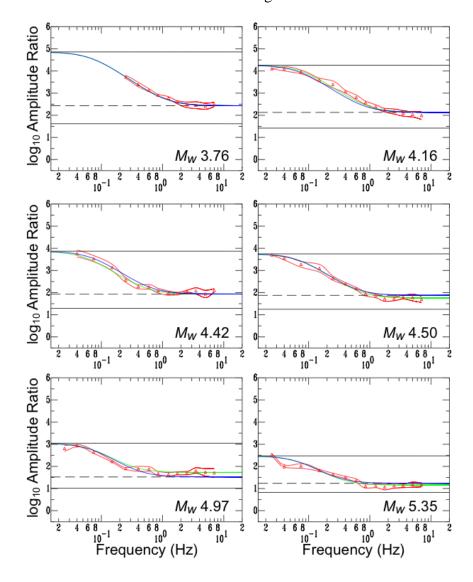
### **Results**

Hector Mine Processing Example

Next, we turn our attention to local and regional recordings of the  $M_w$  7.0 Hector Mine mainshock and 6 aftershocks ranging between  $M_w$  3.7 and 5.4. In this case we consider 6 broadband stations ranging between ~60 and 700 km: GSC, PFO, MNV, CMB, TUC, and ELK. All the events have independent regional seismic moment estimates from full waveform inversion by *G. Ichinose* [pers. comm., 2006] using the methodology outlined in *Ichinose et al.*, [2003]. For consistency, we used the regional estimates of  $M_w$  to avoid any biases since they

were all computed using the same method, velocity model, and station distribution. For the mainshock, we use an  $M_w$  of 7.0 following the results of G. Ichinose [pers. comm., 2006].

Using all 6 stations, we formed the average spectral ratio between the mainshock and each of the aftershocks, then grid-searched using equation 1 and 3 assuming that the reference moment corresponded to an  $M_{\rm w}$  5.0 event and the reference apparent stress was varied between 0.5 and 10 bars. As observed for San Francisco Bay Area events, the coda spectral ratios for Hector Mine events were very stable, with average standard deviations less than 0.1 for all frequencies. Figure 1 shows all 6 ratios assuming both simultaneous model fits (blue lines) and individual ratio fits (green lines). Given the variance of the individual fits, it is preferable to use the simultaneous fit results. In all cases the high frequency asymptote is significantly above the theoretically predicted value. This is consistent with a break in self-similarity where e is between 0.5 and 1.0, and it is inconsistent with a standard self-similar Brune [1970] style omega-square model. What is striking is the low data scatter when compared against results from a conventional direct wave method shown in Figure 1.

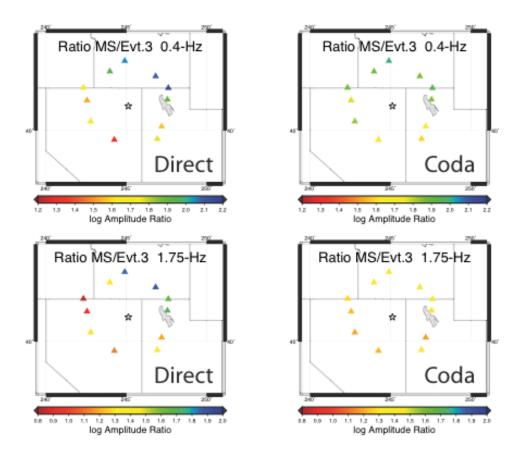


**Figure 1.** Spectral ratios for the Hector Mine mainshock relative to 6 aftershocks. In each figure, we show the low and high frequency asymptotes from equation 1 as solid black horizontal lines and dashed lines represent the case when p=1.5. The average spectral ratio for each band is shown by red triangles and red lines represent +/- 1 σ. Blue lines represent the best fitting MDAC spectral ratios that fit all 6 ratios simultaneously, whereas the green lines represent the case when each ratio was fit individually. We find that the best fitting model corresponds to a reference event of  $M_w$  5.0 with an apparent stress of 0.1 MPa and a scaling parameter, ( $\psi$ ), of 0.25, in good agreement with previous studies using independent methods.

The only way to keep the self-similar assumption and match the data is to have a less than omega-squared spectral decay at high frequencies. However, the decay parameter p must be greater than 1.5 to keep the energy finite [e.g., *Walter and Brune*, 1993]. If the high-frequency decay value p were close to 1.5 it would be possible to nearly match the spectral ratios shown as dashed lines in Figure 1. However, given that such shallow falloff is not consistent with most earthquake observations [e.g., *Hough*, 2001] and independent methods [e.g., *Mayeda et al.* 2005; *Walter et al.*, 2006], our preferred interpretation is that the apparent stresses are systematically lower for the aftershocks than the mainshock. If all events have Brune-style spectra with an  $f^{-2}$  fall-off at high frequencies, this implies the corner frequency scaling is steeper than  $f^{-3}$  for self-similar, constant apparent stress scaling. Next, we plot the inferred corner frequency against seismic moment in Figure 3 and note that estimates from both a simultaneous fit and individual fits show a consistent departure from similarity. Finally, our corner frequency estimate from the simultaneous fitting of the ratios is 0.069 Hz, in good agreement with the teleseismic estimate of 0.059 Hz from *Venkataraman et al.* [2002].

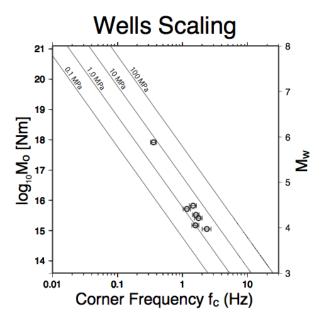
## Insensitivity to Source Radiation Pattern and Directivity

If an average estimate of the source is desired that is stable and free from source and path heterogeneity, then using the coda appears to be ideal, especially when station coverage is sparse. The new coda amplitude ratio methodology employed in this study was first tested on the Hector Mine, CA sequence by Mayeda et al., (2007) and subsequently applied to a number of large magnitude sequences (e.g., Chi-Chi, Taiwan by Mayeda and Malagnini, 2009; San Giuliano, Italy, by Malagnini et al., 2008; and Fukuoka, Japan, by Yoo et al., 2009). Estimates of  $f_c$  were significantly less variable than those derived from either direct-wave amplitude ratios or eGf time-domain deconvolutions, both of which usually suffer from high variance. Figure 2 shows a representative source ratio between the Wells mainshock and an  $M_w$  4.2 aftershock for 0.05 < f < 15.0Hz using the stations surrounding the source region at roughly 200 km distance. The coda is roughly 3-to-4 times more stable than its direct wave counterpart, though their means are virtually identical, confirming that the coda ratios are representative of the S-wave source ratios, but with significantly less variance.

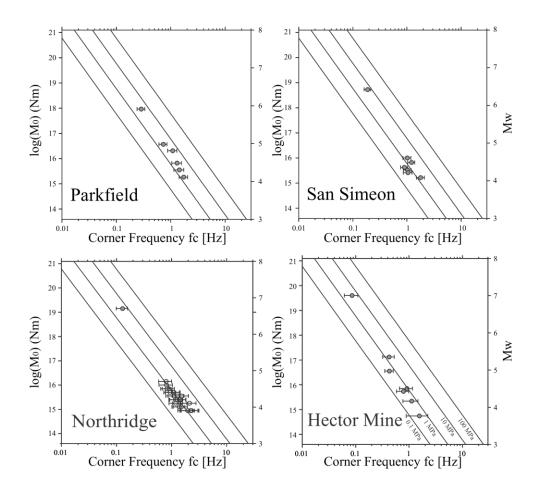


**Figure 2.** Direct and coda wave amplitude ratios between the Wells mainshock (MS) and an  $M_w$  4.2 aftershock (see Mayeda and Malagnini, 2010).

In Figure 3 we plot the  $f_c$  estimates for the Wells, Nevada sequence as a function of  $M_o$  and observe that the mainshock does not appear to trend on the same line of constant  $\Delta \sigma_B$  as the aftershocks, consistent with other studies where the same method has been applied. Though not definitive, when compared with other sequences the current result is consistent with ideas of Mayeda and Malagnini (2009), who suggested that a step-wise increase in the  $f_c$  versus  $M_o$  relation occurs at around  $M_w \sim 5.0$  based on previous studies that used the same ratio methodology. Although the absolute value of the  $f_c$  versus  $M_o$  scaling appears to be region dependent, the scaling change near  $M_w \sim 5.0$  has been observed more often than not. The mainshock  $f_c$  is  $0.36 \pm 0.04$ Hz and is comparable (within the error bounds) to  $f_c$  corresponding to the slip-duration of ~2-3 seconds found by Dreger et al., (2009) and Mendoza and Hartzell (2009) using two finite-fault techniques. Our spectral ratios were also fit using a different methodology and results were in good agreement, within ~30% of our results (R. Gök, pers. comm., 2009).



**Figure 3.** Estimates of  $f_c$  and  $\pm$  1 standard deviation error bars for the Wells mainshock and 6 aftershocks. Lines represent constant  $\Delta \sigma_B$  in MPa.



**Figure 4.** Corner frequency (fc) vs log moment estimates for all four southern California sequences. We see that only the 2004 Parkfield sequence appears to obey self-similarity. The results of the other three sequences are consistent with roughly 15 other large magnitude sequences from around the globe that show a break in scaling at around  $M_w$  5.0-5.5.

## Summary

We have applied the coda ratio methodology outlined in *Mayeda et al.* (2007) to selected large magnitude sequences in southern California. We found that the 2004 Parkfield sequence exhibits self-similarity, whereas the other four sequences, Northridge, Hector Mine, and San Simeon show a clear break in self-similarity. Furthermore, we see subtle absolute differences in the apparent stress and Brune stress drop among the four sequences. As postulated by *Mayeda and Malagnini* (2009), three of the sequences are consistent with a break in similarity occurring near  $M_w \sim 5$  to 5.5 which could signify a change in rupture dynamics between the small and large events. With the recent April 4<sup>th</sup> Baja, Mexico  $M_w$  7.2 event, we also plan on processing this sequence in the near future and including this in our study. A journal paper is in preparation and will be submitted in the near future.

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